65 $\frac{dy}{dx} = x - \frac{1}{x^2}$

a) $x=1 \quad y=2$

$y = \frac{x^2}{2} + \frac{1}{x} + c$

$2 = \frac{1}{2} + 1 + c$

$\frac{1}{2} = c$

$\boxed{y = \frac{x^2}{2} + \frac{1}{x} + \frac{1}{2}}$

$\boxed{(0,\infty)}$

b) $x=-1 \quad y=1$ $\boxed{(-\infty,0)}$

$1 = \frac{1}{2} - 1 + c$ $c = \frac{3}{2}$

$\boxed{y = \frac{x^2}{2} + \frac{1}{x} + \frac{3}{2}}$

c) $y = \begin{cases} \frac{x^2}{2} + \frac{1}{x} + c_1, & x<0 \\ \frac{x^2}{2} + \frac{1}{x} + c_2, & x>0 \end{cases}$

d) $c_1 = 3/2$

$c_2 = 1/2$

Nov 22-9:03 AM

6.2a Integration by Substitution

A change of variables can turn an unfamiliar integral into one that we can evaluate. (The differential matters.)

$$\underbrace{\int f(x)\,dx}_{\text{hard}} = \underbrace{\int g(u)\,du}_{\text{easy}}$$

$\int \sin(x) e^{\cos(x)} dx = \int \cancel{\sin(x)}\, e^u \frac{du}{-\cancel{\sin x}} = -\int e^u du$

let $u = \cos x$

$\frac{du}{dx} = -\sin x$

$du = -\sin x\, dx$

$\frac{du}{-\sin x} = dx$

$= -e^u + c$

$\boxed{y = -e^{\cos x} + c}$

$\frac{dy}{dx} = -e^{\cos x} \cdot (-\sin x)$

$= \sin x\, e^{\cos x}$

$\boxed{\text{let } u = \text{inside of the composite}}$

Nov 30-6:31 PM

$\int x^2 \sqrt{5+2x^3}\,dx = \int \cancel{x^2} \sqrt{u} \frac{du}{\cancel{6x^2}} = \frac{1}{6}\int \sqrt{u}\,du$

$\frac{1}{6}\int u^{1/2}\,du$

$u = 5+2x^3$

$\frac{du}{dx} = 6x^2$

$du = 6x^2\,dx$

$\frac{du}{6x^2} = dx$

$= \frac{1}{6}\cdot\frac{2}{3}u^{3/2} + c$

$= \frac{1}{9}(5+2x^3)^{3/2} + c$

$= \frac{1}{9}\sqrt{(5+2x^3)^3} + c$

we let u = inside of composite

Nov 30-6:40 PM

$\int \cot(7x)\,dx = \int \frac{\cos(7x)}{\sin(7x)}\,dx = \int \frac{\cancel{\cos(7x)}}{u} \frac{du}{7\cancel{\cos(7x)}}$

let $u = \sin(7x)$

$\frac{du}{dx} = 7\cos(7x)$

$\frac{du}{7\cos(7x)} = dx$

$= \frac{1}{7}\int \frac{1}{u}\,du$

$= \frac{1}{7}\ln|u| + c$

$= \frac{1}{7}\ln|\sin(7x)| + c$

we let u = the denominator

Nov 30-6:46 PM

$\int \frac{dx}{\cos^2 2x} = \int \sec^2(2x)\,dx = \int \sec^2 u \frac{du}{2}$

no u-substitution at first

let $u = 2x$

$\frac{du}{dx} = 2$

$\frac{du}{2} = dx$

$\frac{1}{2}\int \sec^2 u\,du$

$\frac{1}{2}(\tan u) + c$

$\frac{1}{2}\tan(2x) + c$

Nov 30-6:48 PM

$\int \cot^2 3x\,dx = \int \csc^2(3x) - 1\,dx = \frac{-\cot(3x)}{3} - x + c$

$\boxed{\sin^2 x + \cos^2 x = 1}$

$\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$

$\boxed{1 + \cot^2 x = \csc^2 x}$

Nov 30-6:50 PM

$\int \cos^3 x\,dx = \int \cos^2 x \cdot \cos x\,dx$

$= \int (1 - \sin^2 x)\cos x\,dx$

let $u = \sin x$ {inside of composite}

$\frac{du}{dx} = \cos x$

$du = \cos x\,dx$

$\frac{du}{\cos x} = dx$

$\int (1 - u^2)\cos x \cdot \frac{du}{\cos x}$

$\int (1 - u^2)\,du = u - \frac{u^3}{3} + c$

$= \sin x - \frac{\sin^3 x}{3} + c$

Nov 30-6:51 PM

Definite Integrals

$\int_0^{\frac{\pi}{3}} \tan x \sec^2(x)\,dx = \int_0^{\sqrt{3}} u\,du = \frac{u^2}{2}\Big|_0^{\sqrt{3}} = \frac{3}{2} - 0$

$u = \tan x \longrightarrow$ $x = 0$ $u = \tan 0 = 0$

$du = \sec^2 x\,dx$ $x = \frac{\pi}{3}$ $u = \tan\frac{\pi}{3} = \sqrt{3}$

is one part the der. of another part? yes

let u =

Nov 30-6:52 PM

$$\int_0^1 \frac{x}{x^2-4}\,dx = \int_{-4}^{-3} \frac{x}{u}\,\frac{du}{2x} = \frac{1}{2}\int_{-4}^{-3} \frac{1}{u}\,du$$

$u = x^2-4$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$x=0 \quad u=-4$

$x=1 \quad u=-3$

$$\frac{1}{2}\ln|u|\Big|_{-4}^{-3}$$

$$\frac{1}{2}\left(\ln|-3| - \ln|-4|\right)$$

$$\frac{1}{2}\left(\ln 3 - \ln 4\right) = \frac{1}{2}\ln\frac{3}{4}$$

Nov 30-7:00 PM